Executive Summary

The Department of Mathematics supports Millikin’s Mission in that the Department works:

1. To prepare students for professional success.
   a. Applied mathematics – we provide core mathematical experiences and a range of application areas to prepare students for work or graduate study.
   b. Mathematics education – we prepare students for the Illinois State Certification Exam, give them experience in teaching, and keep them current on the use of technology in mathematics education.

2. To prepare students for democratic citizenship in a diverse and dynamic global environment.
   a. Applied mathematics - we provide fundamental tools to analyze dynamic events that will inform public policy.
   b. Mathematics education - in a world where political leaders are becoming increasingly numbers driven, we provide the teachers the skills to empower children by enhancing their ability to reason quantitatively.

3. To prepare students for a personal life of meaning and value we help our students develop the intellectual framework, and instill in them the mindset, that will enable them to remain life-long learners. Our students are taught to think rigorously and rationally, and to revel in the sheer pleasure of thinking.

Additionally, the department has specific goals for two of its majors Applied Mathematics, and Mathematics Education. These goals clarify and document the department’s desire to produce highly qualified and successful majors. The University completed the paperwork for the Actuarial Science program to receive VEE credit for applied statistical methods (through 2013), time series (through 2013), corporate finance (through 2014), and economics (through 2014). In 2013 we stated that “A complete assessment of this program including its long-term viability is being developed by Dr. Beck with consolation from the School of Business.” This assessment was not done. The Tabor School of Business was in the middle of redesigning their curriculum and postponed decision on finance / actuarial programs. The mathematics department expects the only assessment criteria beyond those of mathematics major will be to track actuarial exam scores for student who choose this option.

The assessment results for data collected from July 2013- July 2014 constitute the department’s ongoing systemic attempt to quantify student achievement within the department. The results suggest that for students in both Mathematics and Mathematics Education program goals are being met. Additionally, Mathematics Education maintains
NCATE special program accreditation from NCTM. There should be no additional assessment data necessary for the mathematics education major beyond what is collected for the yearly NCATE report completed by Dr. Paula R. Stickles.
Report

Goals

The Department of Mathematics supports the mission of the university in preparing students for professional success, democratic citizenship in a global community, and a personal life of meaning and value. The mission of the department is to produce graduates who achieve the following learning outcome goals:

1. Applied Mathematics
   An applied mathematics major will
   a. be able to integrate and differentiate functions,
   b. be able to express and interpret mathematical relationships from numerical, graphical and symbolic points of view,
   c. be able to read and construct mathematical proofs in analysis and algebra, and
   d. be able to apply mathematics to at least two areas taken from biology, physics, chemistry, economics or computer science.

2. Mathematics Education
   A mathematics education major will
   a. be able to pass the Illinois high school mathematics certification exam,
   b. know in broad terms the history of calculus, algebra, and probability,
   c. have prepared at least 2 lesson plans in mathematics, and
   d. have served as an teaching intern for a member of the mathematics faculty

These goals also reflect a connection to Millikin’s Mission in that the Department works:

4. To prepare students for professional success.
   a. Applied mathematics – we provide core mathematical experiences and a range of application areas to prepare students for work or graduate study.
   b. Mathematics education – we prepare students for the Illinois State Certification Exam, give them experience in teaching, and keep them current on the use of technology in mathematics education.
   c. Computer science – we train students in fundamental programming techniques and theory so that they can learn new technologies in this rapidly changing field.

5. To prepare students for democratic citizenship in a diverse and dynamic global environment.
   a. Applied mathematics- we provide fundamental tools to analyze dynamic events that will inform public policy.
b. Mathematics education- in a world where political leaders are becoming increasingly numbers driven, we provide the teachers the skills to empower children by enhancing their ability to reason quantitatively.

c. Computer science- we provide the skills necessary for students to succeed in an increasingly technological world

3. To prepare students for a personal life of meaning and value we help our students develop the intellectual framework, and instill in them the mindset, that will enable them to remain life-long learners. Our students are taught to think rigorously and rationally, and to revel in the sheer pleasure of thinking.

Snapshot

The Department of Mathematics guides students in the completion of three different majors: mathematics education, applied mathematics and actuarial science. Currently, 22 students are following one of our major programs of study. This is an enrollment increase of 3 from last year.

General Description. The Department of Mathematics includes the disciplines of mathematics and statistics. The department offers mathematic majors with options in Applied Mathematics, Mathematics- Secondary Teaching, and Actuarial Science. Additionally, a minor in Applied Mathematics is offered. Elementary Education majors may take a concentration in mathematics. The curriculum is structured to meet the overlapping needs of students who fall in one or more of the following categories:

- those who plan to become high school mathematics teachers;
- those who intend to pursue graduate work in applied mathematics, computer science, or other related fields; and
- those who will apply mathematics and/or computer science in the natural sciences, social sciences, business or other areas of quantitative studies such as actuarial science.

Additional Comments.

- The three majors offered in the Department share courses and faculty. The applied mathematics and mathematics secondary education majors are particularly entwined with students taking common courses and interacting with the same faculty members. In many respects these two majors cannot be disentangled for analysis.
- Students can earn either the Bachelor of Arts or Bachelor of Science. The choice of B.A. or B.S. depends entirely on the student’s interest in studying a foreign language. There is no distinction in Departmental coursework between the B.A. and B.S. degrees. Therefore, this report will not separate the B.A. from the B.S.
- All fulltime tenure-track members of the Department have doctorate degrees and are tenured. (See Table 1.) The adjunct load has remained constant at 2 FTE for years
with PACE contributing 1 FTE. For fall 2014 this number was reduced by 46 seats (.5 FTE) by increasing all developmental classes to at least 25 from 20.

Description Applied Mathematics. The applied mathematics major is for students interested in immediate employment or further study in applied mathematics or in actuarial sciences. Applied mathematics majors take a minimum of 33 credit hours in mathematics. The core courses and required advanced courses are those specified in *Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004* by the Committee on the Undergraduate Program in Mathematics of The Mathematical Association of America.

Description Mathematics Education. The Mathematics-Secondary Teaching major is a rigorous course of study in mathematics and education. The major has 38 required credit hours in mathematics. Unique among institutions of comparable size we require a mathematics teaching internship experience as part of our program. During this experience the student is paired with a member of the faculty in teaching an undergraduate mathematics course.

Description Actuarial Science Concentration. This option is a rigorous treatment of the mathematics and business skills necessary for a major to enter the workforce as an entry-level actuary. Students who completed this option and all highly recommended courses in business will be prepared to take the first two Actuarial Examinations (1/P and 2/FM) of the Casualty Actuarial Society and the Society of Actuaries. The department is currently working with Tabor School of Business to offer additional course to our majors to prepare them for additional exams. Currently through this corporation, Millikin students can obtain Verification of Educational Experiences (VEE) credit from the Society of Actuaries (SOA) in Applied Statistical Methods (through 2013), Corporate Finance (through 2014), and Economics (through 2014) (see table in appendix).

The Learning Story

Applied mathematics and mathematics education majors follow nearly the same curriculum within the Department. The Department believes that to be a good mathematics teacher one needs to know mathematics. Therefore, the education majors are expected to successfully compete with the applied majors in most of their mathematics courses. The program assumes entering students can start with calculus the fall of their freshmen year. Additionally, education majors are advised to have completed the core of their mathematics courses by the spring of their junior year so that they are prepared for the state certification examination that must be passed prior to being placed for student teaching.

The applied mathematics curriculum focuses on the integration of mathematical theory and mathematical practice. Our majors learn concepts and techniques appropriate for actuarial science, ecological modeling, engineering, numerical analysis, and statistical inference. We assume that most of our applied mathematics major will seek employment in commerce or industry, but the curriculum also prepares them for post-graduate work in mathematics.

The current curriculum maps are included as Appendix 1-2.
Assessment Methods

All students are required to pass the Millikin mathematics placement exam or MA 098 prior to receiving credit for a QR course or receive an equivalent math ACT sub-score (22). In the past, the Department expected our majors to score an ACT math sub-score of 28 or higher or a placement score of 5 (the suggested score for placement into Calculus I). The Department now tests all students wanting to take Calculus with the Millikin Calculus readiness exam and students are placed by the score obtained on the exam. Students are assessed within our programs in numerous ways: course exams, problem sets, and written and oral demonstrations. Additionally, the Department requires every student in Mathematics Education to complete an internship. Written evaluations from these experiences including evaluation by the students’ supervisors are kept. Mathematics Education majors take and pass the state certification examination and submit to a portfolio review. Mathematics majors lead a graduate school like seminar their last semester or an individualized project.

Assessing the Mathematics Major Goals

A mathematics major will

1. be able to integrate and differentiate functions,

All Mathematics majors are required to take and pass both Calculus I and Calculus II to graduate with a Mathematics degree. It is the consensus of the department that it would not be possible to pass these two courses without the ability to integrate and differentiate functions. Therefore, verifying the completion of these two courses by all Mathematics majors will assess fulfillment of this goal. Additionally, the department chair will collect copies of all Calculus I and Calculus II final exams each semester to verify the assertion that integration and differentiation of functions was necessary to pass the exams.

a. In the spring of 2014 the department chair collected copies of all Calculus I and II final exams. The instructors for each course were asked to verify that no student could pass the exam without having knowledge how to integrate and differentiate functions. The department chair then independently verified this conclusion. The collected data in being maintained by the departmental chair and is included at the end of this document.

2. be able to express and interpret mathematical relationships from numerical, graphical and symbolic points of view,

All Mathematics majors are required to take and pass Discrete Mathematics, Differential Equations, and Numerical Analysis. It is the consensus of the department that it would not be possible to pass these three courses without the ability to express and interpret mathematical relationships from numerical, graphical and symbolic points of view. Therefore verifying the completion of these courses by all Mathematics majors will assess fulfillment of this goal. Additionally, the department chair will collect copies of all Discrete Mathematics, Differential Equations, and Numerical Analysis final exams.
each semester to verify the assertion that expressing and interpreting mathematical relationships from numerical, graphical and symbolic points of view was necessary to pass the exams.

a. See attached final exams and reviews of these finals by the individual faculty members.

3. be able to read and construct mathematical proofs in analysis and algebra, and

All Mathematics majors are required to take and pass Discrete Mathematics, Calculus III and Linear Algebra. It is the consensus of the department that it would not be possible to pass these three courses without the ability to read and construct mathematical proofs in analysis and algebra. Therefore verifying the completion of these two courses by all Applied Mathematics majors will assess fulfillment of this goal. Additionally, the department chair will collect copies of all Discrete Mathematics, Calculus III and Linear Algebra final exams each semester to verify the assertion that reading and constructing mathematical proofs in analysis and algebra was necessary to pass the exams.

a. Discrete Mathematics, Calculus III and Linear Algebra were all offered this year. A copy of the final exams from Calculus III and Linear Algebra are attached. A review of these exams support the contention that it would not be possible to pass these three courses without the ability to read and construct mathematical proofs in analysis and algebra. See attached final exams and reviews of these finals by the individual faculty members.

4. be able to apply mathematics to at least two areas taken from biology, physics, chemistry, economics or computer science.

All Mathematics majors are required to take Calculus I and II and Discrete Mathematics. The final exams from all sections of these courses will be review by the department chair to ensure that these routinely contain problems from biology, physics, chemistry, economics or computer science. Specifically, physics will be covered in Calculus I; biology, chemistry, and economics in Calculus II, and computer science applications in Discrete Mathematics.

a. This review was completed and verified that the exam contained appropriate problems involving biology, physics, chemistry, economics or computer science. All final exams for these courses are attached. Again, see attached final exams and reviews of these finals by the individual faculty members.

Assessing the Mathematics Education Major Goals

A mathematics education major will

1. be able to pass the Illinois high school mathematics certification exam,

The department chair will verify that each Mathematics Education major has
passed the state certification exam prior to student teaching. Additionally, the chair will note and analyze the subject area sub scores on an ongoing basis to determine the need for curricular change.

a. All students passed the state exam!
b. The program is nationally accredited!!

2. know in broad terms the history of calculus, algebra, and probability,
   All Mathematics Education majors are required to take and pass Mathematics History to graduate with an Mathematics Education degree. It is the consensus of the department that it would not be possible to pass this course without knowing in broad terms the history of calculus, algebra, and probability. Therefore verifying the completion of this course by all Mathematics Education majors will assess fulfillment of this goal. Additionally, the department chair will audit the Mathematics History syllabus each semester to verify the assertion that the assignments cover the history of calculus, algebra, and probability. Samples of student work will also be collected by the instructor for chair evaluation.

3. have prepared at least 2 lesson plans in mathematics, and
   All Mathematics Education majors will be required to submit 2 graded lesson plans to the department teaching supervisor prior to student teaching. These lesson plans may come from a variety of courses; MA 425 Teaching Secondary and Middle School Mathematics, MA 471 Mathematics Internship, or any other education course that required the completion of a mathematics lesson plan.
   a. Lesson plans for MA 425 and MA471 were collected and reviewed by the department. Dr. Paula R. Stickles has taken over this assessment.

4. have served as a teaching intern for a member of the mathematics faculty
   In support of this goal, all Mathematics Education majors are required to take and pass the departmental teaching internship MA 471 to graduate with a Mathematics Education degree. The departmental chair will collect and analyze the end of course reflection required for this internship to determine the effectiveness of the experience.
   a. All secondary mathematics majors taking MA 471 were required to complete an end of course reflection. These reflections were reviewed by Dr. Paula R. Stickles and she has taken over this assessment.

Assessing the Actuarial Science Major Goals.
An assessment program for the new actuarial science track is also under development. Currently there are 6 students in the program. We are waiting to review the new Tabor School requirements before proceeding.
Analysis of Assessment Results

The assessment data collected for 2013-2014 constitutes the department’s second systemic attempt to quantify student achievement within the department. The results suggest that for students in both Mathematics and Mathematics Education program goals are being met. Assessment of the Actuarial Science program will be delayed until enrollment increases.

Review of 2013-2014 Improvement Plans

- In 2012-2013 all developmental mathematics courses to pass fail. We believe this will improve student performance as they move into QR classes and beyond.
  - As of fall 2013 all developmental courses will be P/F.
    - The department is currently doing data analysis on the grades received by these students.
- In 2012-1213 Obtain funding for a fulltime instructor position to work teach developmental mathematics and PACE course.
  - By the end of spring 2014 we hired a joint appointment to teach ½ time developmental mathematics and d1/2 time PACE advisor.
  - With this appointment the department is still 1 FTE short for covering classes.
- Change QR requirements to continue to reduce the number of mathematics courses students have to take if their major is not mathematics intensive.
  - 2012-2013 We lowered the ACT math score for some QR course to 22. This reduced the number of students needed developing mathematics prior to QR.
  - 2013-14 We approved new QR options for Fine Arts majors.

2013-2014 Improvement Plans Results

- Finish remodeling the mathematics department’s computer lab into a small classroom space.
  - Completed
- Implement an on-line-traditional hybrid model of the department’s first two developmental courses.
  - Completed
- Develop more options for students with high mathematics ACT scores to receive QR credit without taking additional classes.
  - Holding until a larger curriculum overview is done.
- Push for an instructor position for either PACE or developmental traditional faculty.
  - ½ completed
- Develop a new advising form for non-majors to include automatics placement and course registration for students needing to enroll in MA087 in the fall of their freshman year.
  - Completed for edge required students.

2014-2015 Improvement Plans

- Analyze data from all developmental mathematics classes to determine if raising the limit to 25 from 20 has negatively impacted student performance.
- This reduces the department’s FTE need by ½ position (18 developmental classes, or an increase of 90 seats).
Student Publications and Presentations  
Department of Mathematics  
2010-2014

**Peck, H.** Accepted to the Research Experience for Undergraduates in combinatorics at the University of Minnesota, Minneapolis, MN. One of twelve participants in a summer program completing a research project in combinatorics (June-August 2014)

**Spaw, J.** Accepted to the Pacific Undergraduate Research Experience in Mathematics at the University of Hawaii, Hilo, HI. One of twelve participants in a summer program completing a research project in factorization theory (June-July 2014)


**Peck, H.** Conference Presentation. *Irreducible Divisor Graphs with Respect to Tau-Relations*, Western Kentucky University Mathematics Symposium, Bowling Green, KY (November 2013)


**Peck, H.** Accepted to the Summer Mathematics Institute at Cornell University, Ithaca, NY. One of twelve participants in a summer program learning algebra and completing a research project (June-July 2013)

**Peck, H.** Summer Undergraduate Research Fellowship, Millikin University. One of five recipients. (Summer 2012)

**Bloome, L.** Accepted to the Summer Mathematics Institute at Cornell University, Ithica, NY. One of twelve participants in a summer program learning analysis and completing a research project (June-July 2012)

**Bloome, L.** Conference Presentation. *Connections between Central Sets and Cut Sets in Zero-Divisor Graphs of Commutative Rings*, Rose-Hulman Undergraduate Mathematics Conference, Terre Haute, IN, twenty minutes. Recognized as one of the five best talks of the conference. (April 2012)
Buhrmann, J. Conference Presentation. The U.S. Life Insurance Industry: Time Series Analysis, Rose-Hulman Undergraduate Mathematics Conference, Terre Haute, IN, twenty minutes. Recognized as one of the five best talks of the conference. (April 2012)

Perkins, M. Conference Presentation. The Predicted Success Rate in Lower 10 Percent of Accepted Students, Rose-Hulman Undergraduate Mathematics Conference, Terre Haute, IN, twenty minutes. Recognized as one of the five best talks of the conference. (April 2012)

Woods, M. Conference Presentation. Good or Bad: Lowering Entrance Standards, Rose-Hulman Undergraduate Mathematics Conference, Terre Haute, IN, twenty minutes. Recognized as one of the five best talks of the conference. (April 2012)


Morin, M. Conference Presentation. Formalizing Course Materials for a Quantitative Reasoning Course, University of Dayton Undergraduate Mathematics Day, Dayton, OH, fifteen minutes (November 2011)


Stickles, P., **Helding, C., and Smith, B.** Conference Presentation. Authentic Teaching Experiences in Secondary Mathematics Methods Courses. Annual Meeting of the National Council of Teachers of Mathematics. Indianapolis, IN, sixty minutes (April 2011)

**Bloome, L.** Conference Presentation. Compressed Zero-divisor Graphs of Finite Commutative Rings, Rose-Hulman Undergraduate Mathematics Conference, Terre Haute, IN, twenty minutes (March 2011)

**Luciano, G.** Conference Presentation. Using Data Mining to Determine Academic Success in College, Rose-Hulman Undergraduate Mathematics Conference, Terre Haute, IN, twenty minutes (March 2011)


**Bloome, L.** Conference Presentation. Compressed Zero-divisor Graphs of Finite Commutative Rings, Millikin Undergraduate Mathematics Research Conference, Decatur, IL, twenty minutes (November 2010)

**Luciano, G.** Conference Presentation. Using Data Mining to Analyze Admissions Data, Millikin Undergraduate Mathematics Research Conference, Decatur, IL, twenty minutes (November 2010)


<table>
<thead>
<tr>
<th>Faculty</th>
<th>Highest Degree</th>
<th>Rank</th>
<th>Tenure Status</th>
<th>Year Hired</th>
<th>Specialty Field</th>
<th>Courses taught</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daniel Miller</td>
<td>Ph.D.</td>
<td>Professor</td>
<td>Tenured</td>
<td>1997</td>
<td>Mathematics Education, Geometry, Educational Technology.</td>
<td>Teaching Methods, Precalculus, Geometry, Remedial Algebra</td>
</tr>
<tr>
<td>Paula Stickles</td>
<td>Ph.D.</td>
<td>Associate Professor</td>
<td>Tenured</td>
<td>2006</td>
<td>Problem Solving/Posing, Mathematical Modeling</td>
<td>Secondary Methods, Calculus, Mathematics Content for Elementary Teachers</td>
</tr>
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## Curriculum Matrix
### Mathematics

<table>
<thead>
<tr>
<th>Goal 1: be able to integrate and differentiate functions.</th>
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<tbody>
<tr>
<td>Goal 2: be able to express and interpret mathematical relationships from numerical, graphical and symbolic points of view.</td>
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<tr>
<td>Goal 3: be able to read and construct mathematical proofs in analysis and algebra.</td>
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<tr>
<td>Goal 4: be able to apply mathematics to at least two areas taken from biology, physics, chemistry, economics or computer science.</td>
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### Appendix 2

**Curriculum Matrix**  
**Mathematics Education**

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<th>MA 1</th>
<th>MA 2</th>
<th>MA 3</th>
<th>MA 4</th>
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<td>Goal 1</td>
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<td>Goal 2</td>
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<td>Goal 3</td>
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<td>Goal 4</td>
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<th>MA 3</th>
<th>MA 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required Course</td>
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<td></td>
</tr>
<tr>
<td>Elective Courses (Two-required)</td>
<td></td>
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</tbody>
</table>

Goal 1: A mathematics education major will be able to pass the Illinois high school mathematics certification exam.

Goal 2: A mathematics education major will know in broad terms the history of calculus, algebra, and probability.

Goal 3: A mathematics education major will have prepared at least 4 lesson plans.

Goal 4: A mathematics education major will have served as a teaching intern for a member of the mathematics faculty.
Goal: An applied mathematics major will be able to integrate and differentiate functions.

Assessment of goal:

Differentiation: Of the 17 problems on this final exam, problems 3 – 7, 5, 6, 7, 11, 13, 14, and 15 either explicitly or implicitly required the students to take a derivative of some function in order to be able to solve the problem. Problem 1 required the students to understand the definition of the derivative. Problem 7 required the students to connect the first derivative of a function with the function increasing or decreasing and to connect the second derivative with the concavity of the function. Problem 11 required the students to apply differentiation techniques without having an explicitly stated function. Problem 13 required students to connect the derivative to optimizing a quantity given certain restrictions. Problem 14 required students to connect the derivative to a change in quantities with respect to time (related rates).

Integration: Of the 17 problems on this final exam, problems 2, 8, and 9 either explicitly or implicitly required students to integrate some function in order to be able to solve the problem. Problem 2 on the non-calculator part required the students to understand the definition of the definite integral to obtain the exact value of the definite integral. The remaining problems either explicitly or implicitly required students to integrate some function in order to be able to solve the problem.

As nearly every problem on this final exam involved either differentiation or integration (or both), it would be impossible for a student to pass this exam without knowing how to differentiate or integrate functions.

Goal: An applied mathematics major will be able to apply mathematics to at least two areas taken from biology, physics, chemistry, economics, or computer science.

Assessment of Goal: Problem 16 dealt with estimating integrals from a table of values; in particular. Since science students will be making inferences using experimental data, the ability to estimate derivatives and integrals from a table of values will be extremely useful. Problem 13 required students to determine the minimum value of some physical quantity. Though this particular problem did not explicitly bring in physics or chemistry per se, the technique required to solve this problem does occur in solving problems in physics and chemistry, and therefore, students who successfully completed this problem have learned a technique they can use to solve application problems in physics and chemistry. Also, problem 14 involved differentiation to determine the rate of change of a physical quantity with respect to another physical quantity, which is a topic from physics.
Assessment of MA 303 Final Exam for Spring 2014

Goal: An applied mathematics major will be able to read and construct mathematical proofs in analysis and algebra.

A mathematics education major will be able to pass the Illinois high school mathematics certification exam.

Assessment of goal:

This course addresses the following Illinois State Board of Education and NCTM Content Standards.
Standards for Teachers of Mathematics: 8B4, 8F6, and 9F3.
NCTM Indicators: 2.1-2.4, 3.1-3.3, 4.1-4.3, 6.1, 9.7-9.9, 10.2, 10.5, and 11.5.

Further, this course addresses Subarea III of the Illinois Certification Testing System Mathematics (115) exam. (http://www.icts.nesinc.com/PDFs/IL_field115_SG.pdf) An examination of the final exam will show that these indicators and standards have been addressed.

Problems 4, 5, 9–119 require students to construct proofs, while the rest of the exam requires students to understand the concepts presented in proofs in order to complete the problems. Therefore, students must be successful in reading and constructing mathematical proofs in algebra in order to pass this exam.
Assessment of MA240 Final Exam for Spring 2014

Goal: An applied mathematics major will be able to integrate and differentiate functions.

Assessment of goal: Problems 1-6, and 8-12 of the final exam required the students (directly or indirectly) to integrate or differentiate a function. Therefore, it is necessary for students to be able to integrate and differentiate functions in order to pass the final exam.

Goal: An applied mathematics major will be able to apply mathematics to at least two areas taken from biology, physics, chemistry, economics or computer science.

Assessment of goal: Problem 12 required students to apply calculus to solve an application problem. That problem is closely related to chemistry. However, the techniques used can be applied to a number of fields. Therefore, students who successfully complete this problem will be able to apply mathematics to at least two areas taken from biology, physics, chemistry, economics or computer science.
MA240 - Final Exam - Spring 2014

Je m'appelle ____________________________

1. (8 points each) Let \( R \) be the region bounded by the curves \( y = \frac{x^2 + 2x^3 - 8x}{3} \) and \( y = \sin(x) \). Note that these two functions intersect where they cross the \( x \)-axis.

   \[ \text{Diagram:} \]

   (a) Set up, but do not evaluate, an integral (or integrals) that represents the area of \( R \).

   (b) Set up, but do not evaluate, an integral (or integrals) that represents the volume of the solid with a base that is the part of \( R \) to the right of the \( y \)-axis and has square cross sections when taken perpendicular to the \( x \)-axis.

   (c) Set up, but do not evaluate, an integral (or integrals) that represents the volume of the solid generated by revolving the part of \( R \) to the left of the \( y \)-axis about the line \( y = -2 \).
Goal: An applied mathematics major will be able to integrate and differentiate functions.

Assessment of goal: All problems on the final exam required the students (directly or indirectly) to integrate or differentiate a function. Therefore, it is necessary for students to be able to integrate and differentiate functions in order to pass the final exam.

Goal: An applied mathematics major will be able to apply mathematics to at least two areas taken from biology, physics, chemistry, economics or computer science.

Assessment of goal: While not explicitly application problems, problems 6-9 required using theorems that have many applications to physics.
Let me be your ruler. You can call me ________________________.

Directions: Answer the following questions. Show ALL work. An answer with no work receives NO credit.

1. Consider the vector field

$$\mathbf{F}(x, y, z) = (2xe^{x^2\sin y - \ln z})i + \left(\frac{2}{1 + \sqrt{y}}\right)j + \left(x^2e^{x^2\sin y + \tan^{-1}y - \frac{2}{z} + \sec z \tan z}\right)k$$

(a) (6 points) Show that $\mathbf{F}$ is conservative.

(b) (10 points) Find a function $f$ such that $\mathbf{F} = \nabla f$. 
Assessment of MA440 Final Exam for Fall 2013

Goal: An applied mathematics major will be to read and construct mathematical proofs in analysis and algebra.

Assessment of goal: Advanced calculus is the first course mathematics majors see in the more abstract area of mathematics known as analysis. A quick perusal of the final for this course will demonstrate to the reader that the entire course was devoted to reading and constructing mathematical proofs in analysis.
1. (10 points) TRUE or FALSE?

(a) It is possible to find two functions $f$ and $g$ such that $f$ is integrable with respect to $g$ over $[a, b]$, but $g$ is not integrable with respect to $f$ over $[a, b]$.

(b) Suppose $f$ is Riemann integrable on $[a, b]$, and let $F(x) = \int_a^x f(t) \, dt$ for $x \in [a, b]$. It is possible that there exists $c \in (a, b)$ such that $F'(c)$ does not exist.

(c) A function with a finite number of discontinuities over $[a, b]$ is Riemann integrable over $[a, b]$.

(d) Let $f : [0, 1] \rightarrow \mathbb{R}$ be bounded, and define $c_n = \frac{1}{n} \sum_{k=1}^{n} f\left(\frac{k}{n}\right)$ for all $n \in \mathbb{N}$. If $\{c_n\}$ converges, then $f \in R(x)$ on $[0, 1]$.

(e) If $f^2$ is Riemann integrable over $[a, b]$, then $f$ is Riemann integrable over $[a, b]$.

(f) A function with infinitely many discontinuities cannot be Riemann integrable.

(g) If $f$ is Riemann integrable on $[a, b]$ and if $\varepsilon > 0$, then provided the mesh of the partition is small enough, we can use whatever marking of the partition we wish to approximate the value of $\int_a^b f \, dx$ to within $\varepsilon$ of its true value.

(h) If $P$ and $Q$ are partitions of $[a, b]$ with $P \subseteq Q$, then $L(Q, f) \leq U(P, f)$.

(i) If $g'$ exists and if $f$ is integrable with respect to $g$, then $\int_a^b f \, dg = \int_a^b f g' \, dx$.

(j) Every song's like gold teeth, grey goose, trippin' in the bathroom.

2. (10 points) Let $f(x) = \begin{cases} a, & 0 \leq x < 2 \\ b, & x = 2 \end{cases}$. Compute $\int_0^1 f \, dx$ and $\int_0^4 f \, dx$. Is $f$ integrable on $[0, 1]$?

Why or why not?

3. (15 points) Answer the following.

(a) Let $f : [a, b] \rightarrow \mathbb{R}$ be bounded. Then $f \in R(x)$ on $[a, b]$ if and only if for each $\varepsilon > 0$ there is a partition $P$ such that $U(P, f) - L(P, f) < \varepsilon$.

(b) Let $f \in R(x)$ on $[a, b]$ where $f(x) \geq 0$ for all $x \in [a, b]$. Prove $\sqrt{f} \in R(x)$ on $[a, b]$.

4. (15 points) Answer the following.

(a) Prove the Differentiation Theorem for Riemann-Stieltjes integrals: Suppose that $f$ is continuous on $[a, b]$ and that $g$ is increasing on $[a, b]$. If $g'(c)$ exists, where $c \in [a, b]$, then $F'(c) = f(c)g'(c)$, where $F(x) = \int_a^x f \, dg$.

(b) Suppose $f$ and $g$ are continuous on $[a, b]$ with $\int_a^b f \, dx = \int_a^b g \, dx$. Prove there exists $c \in (a, b)$ such that $f(c) = g(c)$.

(OVER)
Part I. Something Old

Subpart A. Calculations (50 points)

1. 
   a. Given that $p$ is true, $q$ is true, and $r$ is false. Determine the
      truth value of $\neg(p \land q) \leftrightarrow (p \rightarrow r)$.
   
   b. What truth value for $x$ would make the statement
      $\neg(y \rightarrow x) \leftrightarrow (\neg x \land \neg z)$ true for all truth values of $y$ and $z$?

2. Write the negation of the converse of the proposition “If the Daleks
   invade the solar system, then the humans will be exterminated.”

3. Let $U = \{0, 1, 2, 3, 4, 5, 6\}$, $A = \{2, 4\}$, and $B = \{1, 2, 3, 4\}$. How many
   different functions $f : B^C \rightarrow A$ are there?
4. Write 3246 as a base 16 numeral.
5. Find integers $m$ and $n$ such that $301m + 115n = 6$.

6. What is the number of distinct permutations of the letters in the word baboon?
7. In how many different ways can 9 diplomats be seated at a roundtable that has only 6 seats?

8. Five people (Amos, Barbara, Claude, Destiny, and Elmer) are lining up for a group photograph. They line up at random and a photograph is taken. Amos and Barbara are married to each other, but Destiny used to be married to Amos. In how many different ways can they line up so that Amos and Barbara are next to each other, but Destiny is not next to Amos?
9. A certain disease is known to be present in 20% of a population of kiwis. There is a test for the disease, but it isn’t perfect. If a kiwi has the disease then the test will return a positive result 98% of the time. On the other hand, if the kiwi doesn’t have the disease then the test will return a positive result 4% of the time. Suppose that a kiwi named Aragorn is tested for the disease and the test returns a positive result. What is the probability that Aragorn has the disease?
Subpart B. Proofs (10 points each)

10. Prove: If $f : A \to A$ and $g : A \to C$ are one-to-one functions, then $g \circ f$ is one-to-one.

11. Let $S$ be the sample space of an experiment and let $E$ and $F$ be disjoint subsets of $S$. Prove that the probability of $E \cup F$ is equal to the sum of the probability of $E$ and the probability of $F$. 
12. Consider the binary relation \( R = \{(a, b) \mid b - a \text{ is evenly divisible by } 4\} \) on the set of integers. Prove that \( R \) is transitive.

13. Let \( f : \{0, 1, 2, 3, 4, 5\} \rightarrow \{0, 1, 2, 3\} \) be the function defined by \( f(n) = n \mod 4 \). Prove that \( f \) is onto.
14. Prove using mathematical induction that $n^2 + n + 4$ is divisible by 2 for all integers $n \geq 1$. 
Part II. Graph Theory (10 points each).

1. Draw a graph with the degree sequence 5,5,4,3,3,2,1 or explain why no such graph exists.

2. Find a subgraph that is isomorphic to $K_{2,3}$ or explain why no such subgraph exists.
3. Prove or disprove that the two graphs are isomorphic.

4. Prove that the number of edges in $K_n$ is $\frac{n(n-1)}{2}$ for all $n \geq 1$. 
5. Suppose that $A$ is the adjacency matrix for a graph $G$ and that

\[
A = \begin{bmatrix}
0 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 \\
\end{bmatrix}.
\]

True or False?

a. $G$ is connected. ______________

b. $G$ has a subgraph isomorphic to $K_{2,1}$. ______________

c. $G$ has an Eulerian subgraph. ______________

6. Find an Eulerian circuit or explain why no such circuit exists.
7. Find a Hamiltonian circuit or prove that no such circuit exists.

8. Prove or disprove: The graph is planar.

10. How many different closed paths of length 8 are there that start and end at vertex B?