Millikin University
Student Learning in the Mathematics and Computer Science Major

By Daniel Miller
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Executive Summary

The Department of Mathematics and Computer Science supports Millikin's Mission in that the Department works:

1. To prepare students for professional success.
    a. Applied mathematics – we provide core mathematical experiences and a range of application areas to prepare students for work or graduate study.
    b. Mathematics education – we prepare students for the Illinois State Certification Exam, give them experience in teaching, and keep them current on the use of technology in mathematics education.
    c. Computer science – we train students in fundamental programming techniques and theory so that they can learn new technologies in this rapidly changing field.

2. To prepare students for democratic citizenship in a diverse and dynamic global environment.
    a. Applied mathematics- we provide fundamental tools to analyze dynamic events that will inform public policy.
    b. Mathematics education- in a world where political leaders are becoming increasingly numbers driven, we provide the teachers the skills to empower children by enhancing their ability to reason quantitatively.
    c. Computer science- we provide the skills necessary for students to succeed in an increasingly technological world

3. To prepare students for a personal life of meaning and value we help our students develop the intellectual framework, and instill in them the mindset, that will enable them to remain life-long learners. Our students are taught to think rigorously and rationally, and to revel in the sheer pleasure of thinking.

Additionally, the department has specific goals for two of its majors Applied Mathematics, and Mathematics Education. These goals clarify and document the department's desire to produce highly qualified and successful majors. In 2008 the department will develop goals for its two new majors (concentrations) Computer Science and Actuarial Science. Additionally the Actuarial Science program will apply for national recognition.

The assessment results for data collected for 2007-2008 constitutes the department's ongoing systemic attempt to quantify student achievement within the department. The results suggest that for students in both Mathematics and Mathematics Education program goals are being meet. It will take at least another year of data ensure that all goals for theses programs are being meet due to the two year rotation of courses in the major and another two years after that to fully integrate the Computer Science and Actuarial Science assessment under development.

Report

Goals

The Department of Mathematics and Computer Science supports the mission of the university in preparing students for professional success, democratic citizenship in a global community, and a personal life of meaning and value. The mission of the department is to produce graduates who achieve the following learning outcome goals:

1. Applied Mathematics

An applied mathematics major will

a. be able to integrate and differentiate functions,
b. be able to express and interpret mathematical relationships from numerical, graphical and symbolic points of view,
c. be able to read and construct mathematical proofs in analysis and algebra, and
d. be able to apply mathematics to at least two areas taken from biology, physics, chemistry, economics or computer science.

2. Mathematics Education

A mathematics education major will

a. be able to pass the Illinois high school mathematics certification exam,
b. know in broad terms the history of calculus, algebra, and probability,
c. have prepared at least 2 lesson plans in mathematics, and
d. have served as an teaching intern for a member of the mathematics faculty

These goals also reflect a connection to Millikin's Mission in that the Department works:

4. To prepare students for professional success.
    a. Applied mathematics – we provide core mathematical experiences and a range of application areas to prepare students for work or graduate study.
    b. Mathematics education – we prepare students for the Illinois State Certification Exam, give them experience in teaching, and keep them current on the use of technology in mathematics education.
    c. Computer science – we train students in fundamental programming techniques and theory so that they can learn new technologies in this rapidly changing field.

5. To prepare students for democratic citizenship in a diverse and dynamic global environment.
    a. Applied mathematics- we provide fundamental tools to analyze dynamic events that will inform public policy.

b. Mathematics education- in a world where political leaders are becoming increasingly numbers driven, we provide the teachers the skills to empower children by enhancing their ability to reason quantitatively.
c. Computer science- we provide the skills necessary for students to succeed in an increasingly technological world

3. To prepare students for a personal life of meaning and value we help our students develop the intellectual framework, and instill in them the mindset, that will enable them to remain life-long learners. Our students are taught to think rigorously and rationally, and to revel in the sheer pleasure of thinking.

Snapshot

The Department of Mathematics and Computer Science guides students in the completion of four different majors: mathematics education, applied mathematics, actuarial science and computer science. Currently, 50 students are following one of our major programs of study. There are 15 students majoring in computer science, 17 in mathematics education, and 19 in applied mathematics. The freshman class contained 6 in computer science, 2 in mathematics education and 2 in applied mathematics. These numbers continue to support the departments assumption that the elimination of computer science as a stand alone major was an error. The Department also serves elementary education students with mathematics concentrations.

General Description. The Department of Mathematics and Computer Science includes the disciplines of mathematics, computer science, and statistics. The department offers majors in Applied Mathematics, Mathematics- Secondary Teaching, mathematics with Computer Science concentration and Mathematics with an actuarial science concentration. Minors are offered in Applied Mathematics and Computer Science. Elementary Education majors may take a concentration in mathematics. The curriculum is structured to meet the overlapping needs of students who fall in one or more of the following categories:

- those who plan to become high school mathematics teachers;
- those who plan to have careers in computer science;
- those who intend to pursue graduate work in applied mathematics, computer science, or other related fields; and
- those who will apply mathematics and/or computer science in the natural sciences, social sciences, business or other areas of quantitative studies such as actuarial science.

Additional Comments.

- The four majors offered in the Department share courses and faculty. The applied mathematics and mathematics secondary education majors are particularly entwined with students taking common courses and interacting with the same faculty members. In many respects these two majors cannot be disentangled for analysis.

- Students can earn either the Bachelor of Arts or Bachelor of Science in any of the four majors offered by the Department. The choice of B.A. or B.S. depends entirely on the student's interest in studying a foreign language. There is no distinction in Departmental coursework between the B.A. and B.S. degrees. Therefore, this report will not separate the B.A. from the B.S.
- All fulltime members of the Department have doctorate degrees and all save one are tenured or tenure-track. (See Table 1.)

Description Applied Mathematics**.** The applied mathematics major is for students interested in immediate employment or further study in applied mathematics or in actuarial sciences. Applied mathematics majors take a minimum of 33 credit hours in mathematics. The core courses and required advanced courses are those specified in *Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004* by the Committee on the Undergraduate Program in Mathematics of The Mathematical Association of America.

Description Mathematics Education. The Mathematics-Secondary Teaching major is a rigorous course of study in mathematics and education. The major has 38 required credit hours in mathematics. Unique among institutions of comparable size we require a mathematics teaching internship experience as part of our program. During this experience the student is paired with a member of the faculty in teaching an undergraduate mathematics course.

Description Computer Science Concentration. The Computer Science concentration will be developed out of the previous computer science major with the inclusion of additional mathematics courses. A complete description and assessment guidelines will be developed in the 2008-2009 academic year by Dr. Rogers and Dr. Rauff.

Description Actuarial Science Concentration. The description will be completed by Dr. Beck in the 2008-2009 academic year as outlined in the JMS project completed by Meghan Sims. This project was shared with the VPAA and the larger Millikin community in the spring of 2008. A unique aspect of the program will be its close tires with the Tabor School of Business.

<u>The Learning Story</u>

Applied mathematics and mathematics education majors follow nearly the same curriculum within the Department. The Department believes that to be a good mathematics teacher one needs to know mathematics. Therefore, the education majors are expected to successfully compete with the applied majors in most of their mathematics courses. The program assumes entering students can start with calculus the fall of their freshmen year. Additionally, education majors are advised to have completed the core of their mathematics courses by the spring of their junior year so that they are prepared for the state certification examination that must be passed prior to being placed for student teaching.

The applied mathematics curriculum focuses on the integration of mathematical theory and mathematical practice. Our majors learn concepts and techniques appropriate for actuarial science, ecological modeling, engineering, numerical analysis, and statistical inference. We assume that most of our applied mathematics major will seek employment in commerce or industry, but the curriculum also prepares them for post-graduate work in mathematics.

Computer Science and Actuarial Science are in a state of transition as program goals and assessments are being developed. These will be ready for the 2009-2010 academic year.

The current curriculum maps are included as Appendix 1-2.

<u>Assessment Methods</u>

All students are required to pass the Millikin mathematics placement exam prior to taking a QR course or receive an equivalent math ACT score. The Department expects our majors to score a 5 (the suggested score for placement into Calculus I). Computer science students are expected to start with Computer Programming I and Discrete Mathematics. Students are assessed within our programs in numerous ways: course exams, problem sets, and written and oral demonstrations. Additionally, the Department requires every student in Computer Science and Mathematics Education to complete an internship. Written evaluations from these experiences including evaluation by the students' supervisors are kept. Mathematics Education majors take and pass the state certification examination and submit to a portfolio review. Applied Mathematics majors lead a graduate school like seminar their last semester.

Assessing the Applied Mathematics Major Goals

An applied mathematics major will

1. be able to integrate and differentiate functions,

   All Applied Mathematics majors are required to take and pass both Calculus I and Calculus II to graduate with an Applied Mathematics degree. It is the consensus of the department that it would not be possible to pass these two courses without the ability to integrate and differentiate functions. Therefore, verifying the completion of these two courses by all Applied Mathematics majors will assess fulfillment of this goal. Additionally, the department chair will collect copies of all Calculus I and Calculus II final exams each semester to verify the assertion that integration and differentiation of functions was necessary to pass the exams.

   a. In the spring of 2008 the department chair collected copies of all Calculus I and II exams. The instructors for each course were asked to verify that no student could pass the exam without having knowledge how to integrate and differentiate functions. The department chair then independently verified this conclusion. The collected data in being maintained by the departmental chair and is included at the end of this document.

2. be able to express and interpret mathematical relationships from numerical, graphical and symbolic points of view,

   All Applied Mathematics majors are required to take and pass Discrete

Mathematics, Differential Equations, and Numerical Analysis. It is the consensus of the department that it would not be possible to pass these three courses without the ability to express and interpret mathematical relationships from numerical, graphical and symbolic points of view. Therefore verifying the completion of these courses by all Applied Mathematics majors will assess fulfillment of this goal. Additionally, the department chair will collect copies of all Discrete Mathematics, Differential Equations, and Numerical Analysis final exams each semester to verify the assertion that expressing and interpreting mathematical relationships from numerical, graphical and symbolic points of view was necessary to pass the exams.

a. In the spring of 2008 Numerical Analysis was not offered. Discrete Mathematics and Differential Equations were offered in the spring of 2008 and both exams are included as attachments. A review of the final exams from these courses support the propositions that it would not be possible to pass these exams without the ability to express and interpret mathematical relationships from numerical, graphical and symbolic points of view.

3. be able to read and construct mathematical proofs in analysis and algebra, and

All Applied Mathematics majors are required to take and pass Discrete Mathematics, Calculus III and Linear Algebra. It is the consensus of the department that it would not be possible to pass these three courses without the ability to read and construct mathematical proofs in analysis and algebra. Therefore verifying the completion of these two courses by all Applied Mathematics majors will assess fulfillment of this goal. Additionally, the department chair will collect copies of all Discrete Mathematics, Calculus III and Linear Algebra final exams each semester to verify the assertion that reading and constructing mathematical proofs in analysis and algebra was necessary to pass the exams.

a. Discrete Mathematics, Calculus III and Linear Algebra were all offered this year. A copy of each final exam is included in the attachments. A review of these exam support the contention that it would not be possible to pass these three courses without the ability to read and construct mathematical proofs in analysis and algebra.

4. be able to apply mathematics to at least two areas taken from biology, physics, chemistry, economics or computer science.

All Mathematics majors are required to take Calculus I and II and Discrete Mathematics. The final exams from all sections of these courses will be review by the department chair to ensure that these routinely contain problems from biology, physics, chemistry, economics or computer science. Specifically, physics will be covered in Calculus I; biology, chemistry, and economics in Calculus II, and computer science applications in Discrete Mathematics.

a. This review was completed and verified that the exam contained appropriate problems involving biology, physics, chemistry, economics or computer science.

Assessing the Mathematics Education Major Goals

A mathematics education major will

1. be able to pass the Illinois high school mathematics certification exam,

   The department chair will verify that each Mathematics Education major has passed the state certification exam prior to student teaching. Additionally, the chair will note and analyze the subject area sub scores on an ongoing basis to determine the need for curricular change.

   a. All students passed the state exam!

2. know in broad terms the history of calculus, algebra, and probability,

   All Mathematics Education majors are required to take and pass Mathematics History to graduate with an Mathematics Education degree. It is the consensus of the department that it would not be possible to pass this course without knowing in broad terms the history of calculus, algebra, and probability. Therefore verifying the completion of this course by all Mathematics Education majors will assess fulfillment of this goal. Additionally, the department chair will audit the Mathematics History syllabus each semester to verify the assertion that the assignments cover the history of calculus, algebra, and probability. Samples of student work will also be collected.

   a. Math History syllabus was collected and reviewed along with student work (see attached)

3. have prepared at least 2 lesson plans in mathematics, and

   All Mathematics Education majors will be required to submit 2 graded lesson plans to the department chair prior to student teaching. These lesson plan may come from a variety of courses; MA 425 Teaching Secondary and Middle School Mathematics, MA 471 Mathematics Internship, or any other education course that required the completion of a mathematics lesson plan.

   a. This is one area of the assessment system that needs more attention. Currently Paula Stickles teaches the MA 425 course and she is not housed within the department and I have not done enough to insure that I receive these lesson plans. The lesson plans are housed within the education departments assessment system because they are also required there. It may be redundant to try to collect them to both assessment documents. Starting fall of 2009 Paula will become a part of the mathematics faculty and I am sure she will want to revisit this part of the departmental assessment system and I look forward to her suggestions for improvement.

4. have served as an teaching intern for a member of the mathematics faculty

    In support of this goal, all Mathematics Education majors are required to take and pass the departmental teaching internship MA 471 to graduate with an Mathematics Education degree. The departmental chair will collect and analyze the end of course reflection required for this internship to determine the effectiveness of the experience.

    a. All secondary mathematics majors taking MA 471 were required to complete an end of course reflection. The chair has reviewed these reflections.

Assessing the Computer Science Major Goals

An assessment program for the new computer science is under development. I expect it to be in place for the fall of 2009.

Assessing the Actuarial Science Major Goals.

An assessment program for the new actuarial science is also under development. I expect it to be in place for the fall of 2009.

Analysis of Assessment Results

The assessment data collected for 2007-2008 constitutes the department's second systemic attempt to quantify student achievement within the department. The results suggest that for students in both Mathematics and Mathematics Education program goals are being meet. It will take at least another year of data ensure that all goals for theses programs are being meet do to the two year rotation of courses in the major and another two years after that to fully integrate the Computer Science and Actuarial Science assessment.

Improvement Plan

Since students are meeting the departmental goals for student learning, we do not plan on making any major changes to our current curriculum. We will however be expanding into actuarial science with a new concentration starting fall 2008. Dr. Beck will be writing the assessment criteria for the new concentration and it will be implemented in the 2010 report.

The department was unsuccessful in hiring an outside candidate for the tenure track applied mathematics position even through we brought in two highly qualified candidates. The problem was salary. After much departmental consideration as to whether or not to try advertising again with the same salary consideration the department decided to move in another direction. Dr. Paula Stickles had been teaching half time for the education department and half time for mathematics. In her pervious position she had taught classes in mathematical modeling and she had sought a move to full time within the department. We looked at this as an opportunity to fill the position with someone we felt was completely qualified and willing to work under the current salary cap. MATHED The only down side is that the department loses half a position which will have to be filled by additional adjunct faculty.

The department applied for NCTM accreditation and it is currently under review.

In the spring of 2008 the computer science major was dropped. It was replaced as a concentration with the mathematics major. Dr Michael Rogers will be writing an assessment program for this new concentration in the spring of 2009 and it will be implemented in the 2010 assessment report.

Table 1. Full time faculty: Mathematics and Computer Science

Faculty	Highest Degree	Rank	Tenure Status	Year Hired	Specialty Field	Courses taught
James Rauff	Ph.D.	Professor	Tenured	1988	Formal Languages, Computational Linguistics, Ethnomathematics.	Discrete Math, Computing Theory, History of Math, Linear Algebra, Calculus, Remedial Algebra.
Randal Beck	Ph.D.	Associate Professor	Tenured	1979	Partial Differential Equations, Statistics.	Calculus, Statistics, Differential Equations.
Daniel Miller	Ph.D.	Associate Professor	Tenured	1997	Mathematics Education, Geometry, Educational Technology.	Teaching Methods, Precalculus, Geometry, Remedial Algebra,
Michael Rogers	Ph.D.	Associate Professor	Tenured	1998	Computer Science.	All courses.
Joe Stickles	Ph.D.	Associate Professor	Tenure-track	2006	Ring Theory.	Calculus, Liberal Arts Mathematics, Abstract Algebra.
Eun-Joo Lee	Ph.D.	Assistant Professor	Tenure-track	2006	Mathematical Statistics.	Statistics, Calculus.
Michael Fearheiley	Ph.D.	Assistant Professor	3-yr term	2000	Chemistry.	Remedial Algebra, Statistics.

Appendix 1

Curriculum Matrix
Applied Mathematics

	MA 140	MA 240	MA 208	MA 301	MA 303	MA 304	MA 320	MA 471	MA 250	MA 302	MA 305	MA 313	MA 314
Goal 1													
Goal 2													
Goal 3													
Goal 4													
	Required Course								Elective Courses (Two-required)				

An applied mathematics major will

Goal 1: be able to integrate and differentiate functions.

Goal 2: be able to express and interpret mathematical relationships from numerical, graphical and symbolic points of view.

Goal 3: be able to read and construct mathematical proofs in analysis and algebra.

Goal 4: be able to apply mathematics to at least two areas taken from biology, physics, chemistry, economics or computer science.

Curriculum Matrix
Mathematics Education

	MA 140	MA 240	MA 208	MA 301	MA 303	MA 304	MA 320	MA 471		MA 250	MA 302	MA 305	MA 313	MA 314
Goal 1														
Goal 2														
Goal 3														
Goal 4														
	Required Course									Elective Courses (Two-required)				

Goal 1: A mathematics education major will be able to pass the Illinois high school mathematics certification exam.

Goal 2: A mathematics education major will know in broad terms the history of calculus, algebra, and probability.

Goal 3: A mathematics education major will have prepared at least 4 lesson plans.

Goal 4: A mathematics education major will have served as a teaching intern for a member of the mathematics faculty.

Detailed Assessment of Selected Courses and Final Exams

Assessment of MA 140 01 Final Exam for Spring 2008

Goal: An applied mathematics major will be able to integrate and differentiate functions.

Assessment of goal:

Differentiation: Of the 12 problems on this final exam, problems 3, 4a, and 6b on the no calculator part, and problems 4 and 5 on the calculator part either explicitly or implicitly required the students to take a derivative of some function in order to be able to solve the problem. Problems 1(a)-(c) on the calculator part required the students to understand the definition of the derivative in order to obtain an estimate, to find an equation of a tangent line, and to use the tangent line to estimate a function vale. Problem 3 on the calculator part required the students to connect the first derivative of a function with finding the absolute maximum and absolute minimum of a function. Problem 2 on the no calculator part required the students to connect the first derivative of a function with the function increasing or decreasing and to connect the second derivative with concavity of the function. Problems 6(a) and 6(d) on the no calculator part required the students to use the graph of the derivative of a function in order to determine behavior of that function. Problem 7 on the no calculator part required the student to understand the connection between Rolle's Theorem and the number of roots of a function.

Integration: Of the 12 problems on this final exam, problems 4(b) and 5(a)-(e) on the no calculator part, and problems 2(a) and 2(b) on the calculator part either explicitly or implicitly required students to integrate some function in order to be able to solve the problem. Problems 1(d) and 1(e) on the calculator part required the students to understand the definition of the definite integral to obtain either the exact value or an estimate of the value of the definite integral. Problem 6(c)-(e) on the no calculator part required the students to interpret the definite integral as (signed) area between a function and the x-axis.

As every problem except one on this final exam involved either differentiation or integration (or both), it would be impossible for a student to pass this exam without knowing how to differentiate or integrate functions.

Goal: An applied mathematics major will be able to apply mathematics to at least two areas taken from biology, physics, chemistry, economics, or computer science.

Assessment of Goal: Problem 1 on the calculator part dealt with estimating derivatives and integrals from a table of values; in particular, students were asked to estimate acceleration and position from a table of velocities, which are topics in physics. Further, since science students will be making inferences using experimental data, the ability to estimate derivatives and integrals from a table of values will be extremely useful. Problem 4 on the calculator part involved differentiation to determine the rate of change of a physical quantity with respect to another physical quantity, which is a topic from physics. Also, problem 5 on the calculator part required students to determine either the maximum value of some physical quantity or. Though this particular problem did not explicitly bring in physics or chemistry per se, the technique required to solve this problem *does* occur in solving problems in physics and chemistry, and therefore, students who successfully completed this problem have learned a technique they can use to solve application problems in physics and chemistry.

12 May 2008 **MA 208 Final Exam** Name: ___________________

Welcome to your MA 208 final exam. Good luck!! The usual rules apply — show your work, be neat, make proofs coherent/cogent, and ask if any question is unclear.

0. Write the following statements in symbolic form, using $\vee$, $\wedge$ ~, and →, using the indicated letters to represent component statements.

Let p = Joan is a politician, s = Joan is smart, l = Joan is a lawyer

a) Joan is not smart, but she is a politician and a lawyer. (2)

b) Joan is neither smart nor a lawyer, but she is a politician (2)

c) If Joan is not a smart lawyer, then she is a politician. (2)

1. Determine whether the indicated pair of statements is logically equivalent. Justify your answer by completing the truth table and providing a modicum of explanation. (6)

$(p \vee q) \wedge r$ and $(p \wedge q) \vee r$

p	q	r	$p \vee q$	$(p \vee q) \wedge r$	$p \wedge q$	$(p \wedge q) \vee r$
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

2. Negate this statement — “The battery is dead or a fuse has blown” — using De Morgan’s laws. (4)

3. a) Write each of the two statements in symbolic form. Let r = “Rob is goalkeeper”; a = “Aaron plays forward”; s = “Sam plays defense”. (6)

• If Rob is goalkeeper and Aaron plays forward, then Sam plays defense.

• if Sam doesn’t play defense then Rob isn’t goalkeeper and Aaron doesn’t play forward.

b) Are the two statements logically equivalent? Justify your answer. (4)

4. Find the converse, inverse, contrapositive and negation for "if today is Tuesday, then this must be Belgium. Express your answer symbolically, using variables t="today is Tuesday", b = "this must be Belgium", and the → symbol.

a) Converse: (2)

b) Inverse: (2)

c) Contrapositive: (2)

d) Negation: (2)

5. Rewrite these statements in if-then form. Express your answer symbolically, using variables w = "Tiger Woods wins a PGA tournament", r = "well rested", s = "shows up", and $\rightarrow$.

a) A necessary condition for Tiger Woods to win a PGA tournament is that he be well rested. (4)

b) A sufficient condition for Tiger woods to win a tournament is that he show up. (4)

c) Tiger Woods will win the tournament only if he shows up. (4)

6. Convert 237 (base 10) to base 2. (4)

7. Convert 205 (base 10) to base 16. (4)

8. Write the following informal statements formally, using symbols $\forall$, $\exists$, $\rightarrow$, R and Z.

a) All real numbers have nonnegative squares (4)

b) Some real numbers are positive (4)

9. Express each statement informally.

a) $\forall$ dinosaurs d, d has a tail (4)

b) $\exists$ computer C | C is solar powered. (4)

10. Find the converse and contrapositive of this statement:

$\forall$ n $\in$ Z, if n is prime then n is odd or n = 2.

a) converse (3)

b) contrapositive (3)

11. Prove that the difference of the squares of any *consecutive* integers is odd. (8)

12. Express 0.467467467... as a rational number. (4)

13. Calculate the following: (4)

a) 35 mod 8

b) 24 div 9

14. Find an explicit formula for this sequence: $1, \frac{-1}{2}, \frac{2}{3}, \frac{-3}{4}, \frac{4}{5}, \ldots$ (3)

15. List the elements of the power set of {a, b, c}. (4)

16. Compute (using the relevant formulae in b) and c))

a) $\prod_{i=2}^{4}(1-\frac{1}{k})$ (4)

b) $\sum_{i=1}^{20} 4^{i}$ (4)

c) $\binom{10}{3}$ (4)

17. Write $\sum_{j=0}^{3}(-1)^{j+1}4^{j}$ in expanded form (5)

18. Let the property P(n) represent the equation $\sum_{i=1}^{n} 4i = 2n^2 + 2n$

a) Verify P(1). (2)

b) State P(k). (2)

c) State P(k+1). (4)

d) Assuming P(k), prove that P(k+1) is true (6)

19. Two standard dice are thrown simultaneously, and the numbers showing face up observed.

a) How many elements are in the sample space of possible outcomes? (4)

b) What is the probability that the numbers showing face up are the same? (4)

c) What is the probability that the numbers sum to 6? (4)

20. How many integers, divisible by 5, are between 15 and 1345, inclusive? (4)

21. How many ways can:

a) the letters in the word “PROJECTS” be arranged, taken 3 at a time? (2)

b) all 8 letters in the word "PROJECTS" be arranged, assuming that "OJ" must stay together, in that order, in honor of the famous breakfast beverage? (4)

22. A license plate consists of 3 letter-digit pairs: letter - digit - letter - digit - letter - digit. For example, C4S 5K2

a) How many distinct license plates are possible? (3)

b) How many license plates are possible in which all the numbers and digits are distinct? (5)

23. An exam consists of 12 questions — 5 multiple-choice, 7 true-false — and students are allowed to choose **9** questions to answer.

a) How many different, 9 question exams are possible? (4)

b) How many different 9 question exams are possible if exactly 3 questions must be multiple choice? (4)

c) How many different 9 question exams are possible if at least 1 question must be true-false?

(4)

24. In a certain discrete math class, three quizzes were given. Out of the 30 students in the class:

15 scored 12 or above on quiz # 1
12 scored 12 or above on quiz # 2
18 scored 12 or above on quiz # 3
7 scored 12 or above on quizzes #1 and # 2
11 scored 12 or above on quizzes # 1 and # 3
8 scored 12 or above on quizzes # 2 and # 3
4 scored 12 or above on all quizzes

a) Draw a Venn Diagram depicting the above. (4)

a) How many scored 12 or above on at least one quiz? (2)

b) How many scored 12 or above on quizzes 1 and 2 but not 3? (3)

25. Let the function (f-g) be defined as (f-g)(x) = f(x) - g(x). If both f and g are *increasing* over R, is (f-g) also increasing? Prove it or give a counterexample. (5)

26. Let S be the set of all strings of a's and b's, and define G(s):S→Z as:

G(s) = number of a's in s - number of b's in s.

a) Is G one-to-one? Prove it or give a counterexample. (4)

b) Is G onto? Prove it or give a counterexample. (4)

27. a) Graph the function $f(x) = \lceil x/4 \rceil$, where f:R→Z. (4)

b) Is f(x) onto? Prove it or give a counterexample. (4)

c) Is f(x) one-to-one? Prove it or give a counterexample. (4)

d) Is f(x) a one-to-one correspondence? Justify your answer. (4)

28. Find a_3, a_4, a_5, and a_6, given that $a_1 = 1$, $a_2 = 1$, $a_k = ka_{k-1} - a_{k-2}$ (4)

29. A single pair of rabbits (male and female) is born at the beginning of a year. Assume the following conditions:

(1) Rabbit pairs are not fertile during their first month of life but thereafter give birth to 3 new male/female pairs at the end of every month.
(2) No rabbits die.

Let r_n = # of pairs of rabbits alive at the end of month n for each integer $n \geq 1$, and let $r_0 = 1$.

a) Find a recurrence relation for r_n (6)

b) How many rabbits will there be at the end of 5 months? (3)

30. Draw a graph if it exists, or if not, explain why it does not:

a) A simple graph with 9 edges and all vertices of degree 3. (4)

b) A graph with 4 vertices of degrees 1, 2, 3, and 4. (4)

c) A $K_{5,2}$ graph. (4)

31. [Do this question *last.*] Prove that there is no least positive rational number. (?)

Congratulations, you have finished your MA 208 Final (and hopefully not vice-versa)!!!!
Enjoy your summer...

MA240 Final Exam Spring 2008

Sign in please: ____

(Every question is worth 10 points)

1. Find the median age of a colony of voles if the probability density function for the ages of the voles (in years) in the colony is given by $f(x) = \frac{1}{576}x^2,\ 0 \le x \le 12$.

2. A region is bounded by the curves $y = x^2 + 2$ and $y = x + 2$. Find the volume of the solid obtaining by revolving this region around the x-axis.

3. A storage bin has a base that is a circle with radius 2 meters $(x^2 + y^2 = 4)$. Cross-sections perpendicular to the base are squares. Write and evaluate the integral whose value gives the volume of this storage bin.

4. Catapult flings a pumpkin from a point 3 meters above the ground at a 30° angle of inclination and with an initial velocity of 97.4 meters/second. Ignoring air resistance, how far does the pumpkin travel from the catapult?

5. A pot of soup is moved from the stove top to a counter top at 12:00 p.m.. The temperature of the room is a constant $70^\circ F$. The initial temperature of the soup was $200^\circ F$. After sitting on the counter for 5 minutes the temperature of the soup cooled to $150^\circ F$. Find the temperature of the soup at 12:10 p.m.

#6-9 refer to the object whose motion in the plane is given by the curve

$$\begin{cases} x(t) = (t-2)^3 - 4(t-2) \\ y(t) = (t-2)^4 - 1 \end{cases}$$

(Assume t is measured in seconds and distance in meters.)

6. Find the slope of the line tangent to curve at $t = 1$.

7. Find the equation of the line tangent to the curve at $t = 1$.

#6-9 refer to the object whose motion in the plane is given by the curve

$$\begin{cases} x(t) = (t-2)^3 - 4(t-2) \\ y(t) = (t-2)^4 - 1 \end{cases}$$

(Assume t is measured in seconds and distance in meters.)

8. Find the speed of the object at $t = 1$ and describe its motion.

9. Find the area enclosed by the curve for $0 \le t \le 4$.

10. An object's motion in the plane is given by the curve $\begin{cases} x(t) = (t-2)^3 - 4(t-2) \\ y(t) = (t-2)^4 - 1 \end{cases}$

A second object's motion in the plane is given by the curve $\begin{cases} x(t) = \dfrac{-3}{4}t \\ y(t) = \dfrac{9}{8}t + \dfrac{1}{8} \end{cases}$

(Assume that time is measured in seconds and distance in meters.)

The two objects start into motion at the same time ($t = 0$) and later collide at the point $\left(\dfrac{-21}{8}, \dfrac{65}{16}\right)$. What is the speed of the faster object at the time of collision?

11. Evaluate: $\int (x+2)e^x dx$ (Show your work.)

12. Evaluate: $\int \cos^3 x \sin^2 x \, dx$. (Show your work.)

13. Evaluate: $\int_{1}^{\infty} \frac{2}{x^2}\,dx$. (Show your work.)

14. Evaluate: $\int \frac{4p}{x^2 - p^2}\, dx,\quad (p > 0)$ (Show your work.)

15. Solve the differential equation. $\dfrac{dy}{dt} = \dfrac{e^{-y}}{t^2}, \quad y(1) = \ln 4$

16. Find the equilibrium points for the competing species model

$$x' = 0.3x - 0.1x^2 - 0.2xy$$
$$y' = 0.2y - 0.2y^2 - 0.2xy$$

17. Use the integral test to decide whether $\sum_{k=1}^{\infty} \frac{2x}{x^2+1}$ converges or diverges.

18. Decide whether the series $\sum_{k=0}^{\infty} \frac{(-1)^k 6^k}{5^k}$ converges or diverges. Explain your answer using one of the convergence tests.

19. Find the radius and interval of convergence for the power series $\sum_{k=0}^{\infty} \frac{(x-2)^k k}{3^k}$

20. Find the Taylor series about $c = 1$ for the function $f(x) = \frac{1}{x^2}$ and determine its interval of convergence.

Insert Cal 3 exam here

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1. (16 points) Given the vectors **a**, **b**, **c** compute the following:

a = 3**i** + 5**j** - 4**k**

b = -6**i** - 2**j** + **k**

c is the vector with head at (2,1,0) and tail at (0,-1, 6)

a) **a** + **b**

b) **a** • **c**

c) **a** x **b**

d) || **a** ||

2. (12 points) Find the angle between the vectors <1,5> and <4,2>

3. (12 points) Find a vector in the opposite direction of <2, -3, -5> with length 9.

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4. (12 points) Find the equation of the plane through the points (1,2,-3), (4,-5,6) and (-9,8,7).

5. (12 points) Find the unit tangent vector, **T**(t), to **r**(t) = <3cos(t), 2sin(t)> at

a) $t = 0$

b) $t = \pi/2$

6. (12 points) Find the equation of the line tangent to the graph of **r**(t) = <t^2, cos(πt), ln(t)> at $t = 1$.

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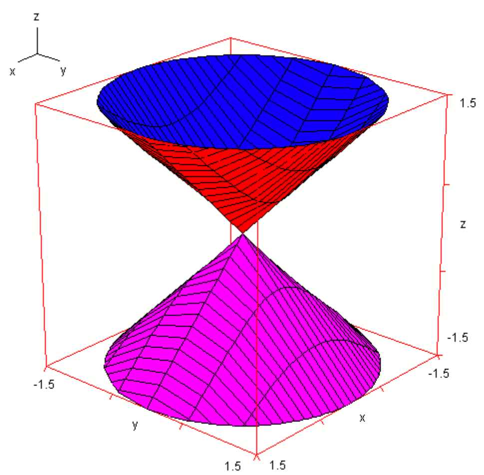
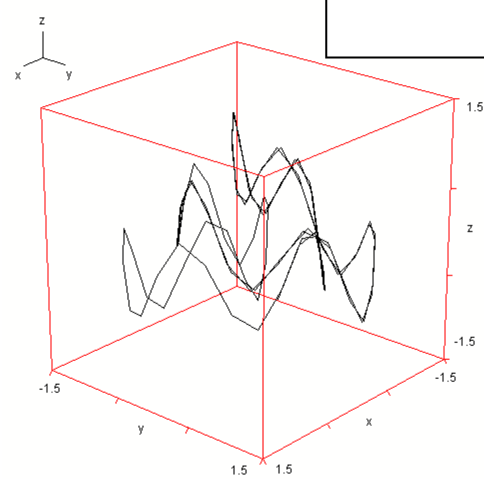
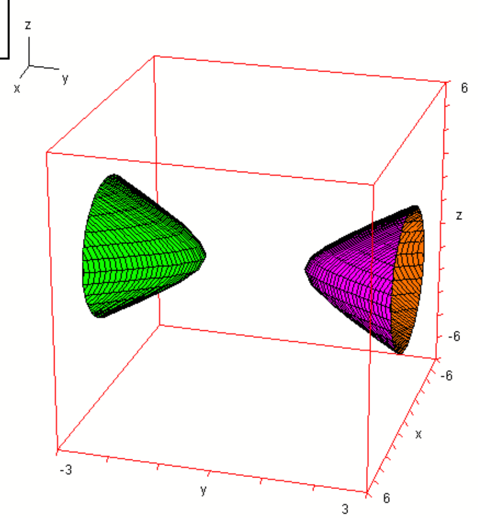
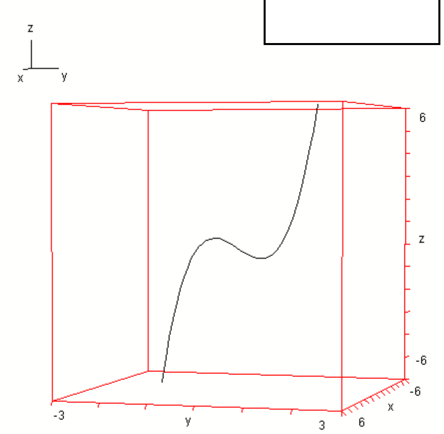
7. (12 points) Match each of the graphs to its function.









a) $z^2 = x^2 - y2$

b) $x^2 + y^2 - z^2 = 1$

e) $r(t) = [\cos(t), \sin(t), \sin(t)]$

f) $r(t) = [t, t, t^3 - t]$

c) $z^2 = x^2 + y^2$

g) $r(t) = [\cos(t), \sin(3t), \cos(5t)\sin(7t)]$

d) $x^2 - y^2 + z^2 = -1$

h) $r(t) = [t, -t, t^2-t]$

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8. (12 points) Demonstrate that the indicated limits do not exist.

a) $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$

b) $\lim_{(x,y)\to(0,0)} \frac{2xy^2}{x^2+y^4}$

9. (12 points) Let f(x,y) = $x\sqrt{x^2+y^2}$. Find the directional derivative of f(x,y) at the point and direction given.

a) P = (3,4) **u** = $\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$

b) P = (-5, 12) **v** = <6, -8>

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10. (12 points) Find and classify all critical points.

a) $f(x,y) = 4 + e^{-2}x - e^{2}y$

b) $f(x,y) = x^2 - 4xy + y^2$

11. (14 points) Integrate.

a) $$\int_{-1}^{1}\int_{1}^{2} 12x^2 y \, dy \, dx$$

b) $$\int_{0}^{2}\int_{y}^{2} x^2 + 3y^2 \, dx \, dy$$

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12. (12 points) Find the maximum and minimum of $f(x,y) = 3x^2 + y^2$ subject to the constraint $x^2 + 2y^2 \le 2$

Assessment of MA 303 01 Final Exam for Spring 2008

Goal: An applied mathematics major will be able to read and construct mathematical proofs in analysis and algebra.

Assessment of goal:

Problems 2, 3, 4, 5(a), 8, and 9 on the take-home portion of the final and problems 6 and 7 on the in-class portion of the final require students to construct algebraic proofs. Since these problems comprise almost half of the final exam, it is necessary for students to be able to read and construct mathematical proofs in algebra in order to pass the final exam.

MA 303 - Linear Algebra - Final Exam - Spring 2008

Directions: Answer the following questions on the paper provided. Please begin each new problem on a separate sheet of paper and only write on one side of the paper. Show all your work. An answer with no work receives NO credit. You may use your calculator to perform matrix operations and row reductions, unless the problem specifically asks you to perform the operation by hand.

1. (6 points each) Find the eigenvalues and bases for the eigenspaces associated with each eigenvalue for the following matrices.

   (a) $\begin{bmatrix} 5 & 3 \\ 1 & 7 \end{bmatrix}$

   (b) $\begin{bmatrix} 5 & 1 & 0 \\ 0 & 5 & 3 \\ 0 & 0 & 2 \end{bmatrix}$

   (c) $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

2. (10 points) For each matrix A in problem 1, find a nonsingular matrix P that diagonalizes A, if possible. If this is not possible, state why. For those matrices which are diagonalizable, compute A^{10} using the diagonalization.

3. An object is traveling downward, and its velocity v (in meters per second) is recorded at various times t (in seconds).

t	1	2	3	4	5
v	-13	-23	-32.5	-42.3	-52

   (a) (6 points) Find the least-squares line that models this data.

   (b) (4 points) Using this model, predict the velocity of the object after 6 seconds.

   (c) (4 points) Using this model, estimate the initial velocity of the object.

   (d) (4 points) Using this model, estimate the acceleration due to gravity. (HINT: What is the "calculus" relationship between velocity and acceleration?)

4. The work W (in ergs) done in stretching a spring x cm beyond its natural length is recorded for several values of x.

x	1	2	3	4	5	6
W	2	7	20	32	47	75

   (a) (6 points) Find the least-squares quadratic that models this data.

   (b) (4 points) Using this model, predict the work done in stretching the spring 7 cm beyond its natural length.

MA 303 - Take-Home Final Exam - Spring 2008

Please read the statement below and sign your name on the line provided. This form should be handed in with your work when you turn it in. If you do not turn in this signed statement, you will not receive credit for this exam. This portion of the exam is worth 100 points and is due at 10:30 am on Tuesday, May 13, 2008. No late examinations will be accepted.

By my signature below, I acknowledge that I have not collaborated with anyone while preparing this examination. I understand that any verbal, written, electronic, or other form of communication with another person (besides Dr. Stickles) in reference to this exam constitutes collaboration. I understand that I may use other written sources (the textbook for the course, my notes, other textbooks in the library, websites, etc.). However, if I use a written source in completion of a problem, I understand that I must cite these sources appropriately and that I must provide more detail than is contained in the written source. If it is determined that I have collaborated with another person, if I used a written source without proper citation, or if I used a written source with proper citation but did not provide more detail than is included in the written source, I understand that I will receive a zero on this exam.

____

Signature

____

Name (printed)

MA 305
Beck **Final Examination**
5/13/8

name ___________________________

Solve 10 of the following 11 problems.
The problem that should not be graded is number __________

1. (15 points) Solve the following ordinary differential equations.

a) $x' = x^{-2}t^{3}$

b) $\dfrac{dy}{dx} = \dfrac{1-xy}{x^2}$

2. (15 points) Solve the following ordinary differential equations.

a) $(t+\ln(t))\dfrac{dx}{dt} = -x - \dfrac{x}{t}$

b) $y' - y = 0$

3. (15 point) Solve the following initial value problems.

a) $x' = 5(x-10)$ $x(0) = 2$

b) $y' = f(x,y)$ where $f(x,y) = \dfrac{x-y^2}{2xy-2y}$ and $y(2) = 3$

4. (15 point) Solve the following initial value problems.

a) $\dfrac{dr}{dt} = 1-r$ $r(0) = -10$

b) $y' + y = e^{-x}$ $y(1) = 2$

5. (15 points) Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference between the object's temperature and the temperature of the environment (assumed to be constant. Thus

$$\frac{dT}{dt} = k(T - T_0)$$

Where $T(t)$ is the temperature of the object at time t, T_0 is the temperature of the environment and k is the proportionality constant.

A slab of molten iron is removed from a blast furnace with a temperature of 1750°C. After 10 minutes, the temperature of the slab is 1650°C. The temperature of the surrounding air is a constant 400°C.

a) What is the temperature of the slab at time t?

b) When is the temperature of the slab 1500°C?

6. (15 points) Solve the following homogeneous equations.

a) $y'' + 3y' - 4y = 0$

b) $2\frac{d^2y}{dt^2} - 8\frac{dy}{dt} + 16y = 0$

7. (15 points) Solve the following homogeneous equations.

a) $\frac{d^4x}{dt^4} - x = 0$

b) $\frac{d^3y}{d\theta^3} - 6\frac{d^2y}{d\theta^2} + 9\frac{dy}{d\theta} = 0$

8. (15 points) Solve the following nonhomogeneous equations.

a) $y'' - 4y = \cos(2x) - \sin(2x)$

b) $x'' + 2x' - 3x = t^2$

9. (15 points) Solve the following nonhomogeneous initial value problem.

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 2x = 6t \qquad x(0) = 1, \frac{dx}{dt}(0) = 4$$

10. (15 points) A spring and mass system had a mass of 2 kg attached to a spring with constant 10. A damping force equal to 8 times the velocity of the mass resists the motion. The mass is displaced 0.02 meters and released.

a) Find the position of the mass x(t) at time t.

b) How many times does the mass reach its equilibrium position?

11. (15 points) Solve the system of first order differential equations.

$$\frac{dx}{dt} = 7x - 3y \qquad x(0) = 75$$

$$\frac{dy}{dt} = 4x - y \qquad y(0) = 70$$

Insert 320 materials